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# The state evolution formulation of teleportation for continuous variables

Junxiang Zhang[1,2], Changde Xie[2], Fuli Li[1,3], Shi-Yao Zhu[1] and M. Suhail Zubairy[4]

[1] *Department of Physics, Hong Kong Baptist University - Hong Kong, PRC*
[2] *The Key Laboratory of Quantum Optics, Ministry of Education - PRC and Institute of Opto-electronics, Shanxi University - PRC*
[3] *Department of Physics, Xi'an jiaotong University - PRC*
[4] *Department of Physics, Texas A&M University College Station - TX 77843, USA*

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**Abstract.** – The quantum teleportation of continuous variables is formulated through the state evolution in the Schrödinger picture from the viewpoint of general quantum-mechnical measurement theory. This method provides the connection between the teleportation and the reversible quantum operations. In this way, the state evolution process of teleportation can be presented clearly step by step, the criterium of teleportation for a coherent state is also deduced.

*Introduction.* – Quantum teleportation has attracted much interest in quantum computation for error correction protocols, optical quantum communication for sending secret messages and superdense coding of information. Hence, we see that it plays an important role in the relatively new, but rapidly growing, field of quantum information science.

The first proposal of quantum teleportation was made by Bennett *et al.* in spin-(1/2) particle systems [1]. Recent experiments [2,3] demonstrated that this effect could be accomplished by teleporting the polarization state of a single photon. More recently, Vaidmann presented a principle to extend the teleportation scheme to the systems with continuous variables [4]. More realistic theories for quantum teleportation of continuous variables in different ways such as in terms of Wigner function, quadrature representation and discrete basis state [5–10] have been developed. This analysis has also been applied to continuous variable entanglement swapping [11–13] and super-dense quantum coding [14]. The experimental realization of teleportation for continuous variables of an optical coherent state was demonstrated [15], where the squeezed lights are exploited to the source of EPR pair. In fact there is no possibility to produce perfect EPR pairs for continuous variables, thus the criterium for evaluating the efficiency of realistic teleportation has to be considered [5, 16–18].

Quantum teleportation is a process of disintegrating a quantum state in one place and then reconstructing it at a remote site. The physical mechanisms behind it are the quantum nonlocal correlation and the Bell-operator measurements. Because of the nonlocal correlation between the EPR pairs [19], when performing joint Bell-operator measurements on the input state and one half of the EPR pair, the other half of the EPR pair automatically collapses into a quantum state which relates to the input teleported state, then, by means of a definite unitary transformation according to the outcome of the Bell measurement, the teleported state is reconstructed. The teleportation can be understood as a reversing quantum operation consisting of the reversible measurements and unitary transformation operation [20, 21]. In this paper we provide an approach in which we analyze the state evolution process of the continuous variable teleportation with the coherent-state representation and apply the scheme of reversible quantum operation in the quantum measurement theory. To our knowledge it has not been presented before. All published papers deal with teleportation process in the $Q$- and $W$-representation [5, 16] or in the Heisenberg picture [7]. A similar discussion using the quantum measurement theory has been given [6, 9], but only the case of perfect two-mode squeezed state is considered. In our calculation there is no any approximation on the squeezing parameter of the two-mode squeezed state, therefore a general expression of teleportation for continuous variables is obtained and the fidelity is deduced directly from the original definition.

*EPR entanglement for continuous variables.* – The first step of teleportation is the preparation of EPR quantum entangled pairs. In the teleportation scheme for continuous variables proposed in ref. [5], the EPR entangled state is a two-mode squeezed vacuum state, which is expressed as [6, 8]

$$\rho_{1,2} = (1-\lambda^2)\sum_{n}^{\infty}\sum_{n'}^{\infty}(-\lambda)^{n+n'}\,|n,n\rangle_{1,2}\langle n',n'|\,, \tag{1}$$

where $\lambda = \tanh r$ $(0 < \lambda < 1)$ is a parameter quantifying the strength of squeezing, the indices $1, 2$ denote the two modes of the squeezed state, $|n, n\rangle$ is the Fock state corresponding to 1 and 2 modes. In the limit $r \to \infty$, *i.e.*, $\lambda \to 1$, the squeezed vacuum state is maximally squeezed, in this case the two-mode squeezed state approaches a simultaneous eigenstate of $\hat{X}_1 + \hat{X}_2$ and $\hat{Y}_1 - \hat{Y}_2$ as described in ref. [4], where $\hat{X}_{1,2} = (\hat{a}_{1,2} + \hat{a}^+_{1,2})$ and $\hat{Y}_{1,2} = -i(\hat{a}_{1,2} - \hat{a}^+_{1,2})$ are the quadrature phase amplitudes for the two modes $a_1$ and $a_2$ of the squeezed state. For the eigenstate, the quantum fluctuations of $\hat{X}_1 + \hat{X}_2$ and $\hat{Y}_1 - \hat{Y}_2$ are equal to zero. This is an analogue of the EPR state with both entanglements between quadrature amplitudes $\hat{X}_1$ and $\hat{X}_2$ as well as between quadrature phases $\hat{Y}_1$ and $\hat{Y}_2$. For the finite squeezing, when the uncertainty product for the variances of the two inferences $\langle\Delta^2\hat{X}_{\rm inf}\rangle\langle\Delta^2\hat{Y}_{\rm inf}\rangle = \langle\Delta^2(\hat{X}_1 + g_x\hat{X}_2)\rangle\langle\Delta^2(\hat{Y}_1 - g_y\hat{Y}_2)\rangle$ is less than the limit of unity associated with the Heisenberg uncertainty relation, the EPR paradox for continuous variables is demonstrated [22, 23]; here $g$ is the scaling factor for minimizing the variances. From eq. (1) we can calculate [22] $\langle\Delta^2\hat{X}_{\rm inf}\rangle\langle\Delta^2\hat{Y}_{\rm inf}\rangle = (1/\cos h2r)^2$, thus once squeezing exists, *i.e.* $r > 0$, the uncertainty product $\langle\Delta^2\hat{X}_{\rm inf}\rangle\langle\Delta^2\hat{Y}_{\rm inf}\rangle < 1$. When $r = 0$ (*i.e.* $\lambda = 0$) we have $\langle\Delta^2\hat{X}_{\rm inf}\rangle\langle\Delta^2\hat{Y}_{\rm inf}\rangle = 1$, that is just the classical limit of EPR entanglement. For the general case the EPR paradox is violated for the minimum uncertainty squeezed states.

*The state evolution for teleportation of continuous variables.* – In this section, we explore how the state evolves in the process of teleportation of continuous variables. For this purpose and more generality, we consider the quantum system in the Schrödinger picture in terms of coherent-state representation. In this case we expand the entangled state in coherent-state basis as

$$\rho_{1,2} = \sqrt{(1-\lambda^2)} \int\int \mathrm{d}^2\mu \mathrm{d}^2\nu e^{-\frac{1}{2}|\mu|^2} e^{-\frac{1}{2}|\nu|^2} e^{-\lambda\mu^*\nu*} \; |\mu\rangle_1|\nu\rangle_2 \otimes \mathrm{h.c.} \tag{2}$$

In the Glauber-Sudarshan coherent-state representation, the input state of the signal mode can be expressed as

$$\hat{\rho}_{\mathrm{in}} = \int \mathrm{d}^2\alpha P(\alpha)|\alpha\rangle_{\mathrm{in}}\langle\alpha| \,, \tag{3}$$

where $|\alpha\rangle_{\mathrm{in}}$ represents a coherent input state, $\alpha$ is a complex variable and $P(\alpha)$ is called the $p$ function, which is a quasi-probability density function in the phase space, and usually is normalized as $\int \mathrm{d}^2\alpha P(\alpha) = 1$.

In the Alice sending station, the density operator of the initial state combining the input unknown state with the EPR pair is

$$\hat{\rho}_0 = \hat{\rho}_{\mathrm{in}} \otimes \hat{\rho}_{1,2} \,. \tag{4}$$

The second step of teleportation is to perform a joint measurement on the input teleported state and a half of the EPR pair. In practice, this process is accomplished by combining the input state with a half of the EPR pair on a 50:50 beamsplitter and then homodyne detecting the quadrature amplitude of one output mode of the beamsplitter and the quadrature phase of the other output mode [15].

As is well known, a 50:50 lossless beamsplitter is described by the unitary operator $\hat{U}$ [24],

$$\hat{U} = e^{\frac{\pi}{4}\left(\hat{a}_{\mathrm{in}}\hat{a}_1^\dagger - \hat{a}_1\hat{a}_{\mathrm{in}}^\dagger\right)} \,; \tag{5}$$

for simplicity, we have ignored the phase shift introduced by the beamsplitter in eq. (5). Equation (5) shows that a lossless beamsplitter is characterized by the generators of the $SU(2)$ Lie group whose input and output relationship in the Heisenberg picture is given by $\hat{a}_{\mathrm{in}}^{\mathrm{out}} = (\hat{a}_{\mathrm{in}} + \hat{a}_1)/\sqrt{2}$ and $\hat{a}_1^{\mathrm{out}} = (\hat{a}_1 - \hat{a}_{\mathrm{in}})/\sqrt{2}$.

In the Schrödinger picture, the density matrix of the output state of the beamsplitter is obtained through the unitary transformation of the density matrix $\hat{\rho}_0$ of the initial state [25, 26]:

$$\hat{\rho}_{\mathrm{BS}} = \hat{U}^\dagger \hat{\rho}_0 \hat{U} \,. \tag{6}$$

Using the decomposition formula of the $SU(2)$ Lie algebra [27],

$$\hat{U}^\dagger = e^{\hat{a}_{\mathrm{in}}^\dagger \hat{a}_1} (\sqrt{2})^{\hat{a}_{\mathrm{in}}^\dagger \hat{a}_{\mathrm{in}} - \hat{a}_1^\dagger \hat{a}_1} e^{-\hat{a}_{\mathrm{in}}\hat{a}_1^\dagger}, \tag{7}$$

and the following relations:

$$(\sqrt{2})^{-\hat{a}_1^\dagger \hat{a}_1}|\gamma\rangle_1 = e^{-\frac{1}{4}|\gamma|^2}|\gamma/\sqrt{2}\rangle_1, \qquad (\sqrt{2})^{\hat{a}_{\mathrm{in}}^\dagger \hat{a}_{\mathrm{in}}}|\alpha\rangle_{\mathrm{in}} = e^{\frac{1}{2}|\alpha|^2}|\sqrt{2}\alpha\rangle_{\mathrm{in}}, \tag{8}$$

we substitute eqs. (4) and (7) into eq. (6) and calculate the density matrix of the output state of the beamsplitter in coherent-state basis:

$$\begin{aligned}\hat{\rho}_{\mathrm{BS}} = {} & \frac{(1-\lambda^2)}{\pi^4} \int \mathrm{d}^2\alpha P(\alpha) \times \\ & \times \iint \mathrm{d}^2\beta \mathrm{d}^2\gamma e^{-\frac{3}{4}|\gamma|^2} e^{\frac{1}{2}|\lambda\gamma|^2} e^{-\frac{1}{2}|\beta|^2} e^{-\frac{1}{2}|\alpha|^2} e^{-\alpha\gamma^*} e^{\beta^*\gamma/\sqrt{2}} e^{\sqrt{2}\alpha\beta^*} |\beta\rangle_{\mathrm{in}}|\gamma/\sqrt{2}\rangle_1|-\lambda\gamma^*\rangle_2 \otimes \\ & \otimes \iint \mathrm{d}^2\zeta \mathrm{d}^2\eta {}_2\langle-\lambda\eta^*|_1\langle\eta/\sqrt{2}|_{\mathrm{in}}\langle\zeta| e^{-\frac{3}{4}|\eta|^2} e^{\frac{1}{2}|\lambda\eta|^2} e^{-\frac{1}{2}|\zeta|^2} e^{-\frac{1}{2}|\alpha|^2} e^{-\alpha^*\eta} e^{\zeta\eta^*/\sqrt{2}} e^{\sqrt{2}\alpha^*\zeta} \,.\end{aligned} \tag{9}$$

In the general case of continuous variable teleportation, the amplitude quadrature $\hat{X}$ of one output beam of the beamsplitter and the phase quadrature $\widehat{Y}$ of the other one are simultaneously measured with two homodyne detection systems. In the Schrödinger picture the measured outcomes correspond to the amplitude quadrature of the input state and the phase quadrature of a half of the EPR pair, that are

$$\hat{X} = (\hat{a}_{\rm in} + \hat{a}_{\rm in}^{\dagger})/2\,, \qquad \widehat{Y} = -i(\hat{a}_1 - \hat{a}_1^{\dagger})/2\,. \tag{10}$$

According to the general theory of quantum-mechanical measurement, the quantum measurement is mathematically described by a positive operator-valued measure (POVM) including a projection operator. Therefore, the positive operator-valued measurement of the two quadrature phases of $X$ and $Y$ are given by [28]

$$\prod_{\rm in}(X) = |X\rangle_{\rm in}\langle X|\,, \qquad \prod_{1}(Y) = |Y\rangle_1\langle Y|\,, \tag{11}$$

where the states $|X\rangle_{\rm in}$ and $|Y\rangle_1$ are the eigenstates of the quadrature components satisfying the completeness relations: $\int_{-\infty}^{\infty} |X\rangle_{\rm in}\langle X|{\rm d}X = \hat{I}$, $\int_{-\infty}^{\infty} |Y\rangle_1\langle Y|{\rm d}Y = \hat{I}$. When the values are measured, the normalized density matrix of the conditional output state of the other half of the EPR pair collapses into

$$\hat{\rho}_2(X,Y) = \frac{{\rm Tr}_{\rm in,1}\{\hat{\rho}_{\rm BS}\prod_{\rm in}(X)\prod_1(Y)\}}{P(X,Y)}\,, \tag{12}$$

where ${\rm Tr}_{\rm in,1}$ stands for the trace operation with respect to the input state and half of the EPR pair. $P(X,Y)$ is the probability distribution of the measured results:

$$P(X,Y) = {\rm Tr}_2{\rm Tr}_{\rm in,1}\{\hat{\rho}_{\rm BS}\prod_{\rm in}(X)\prod_1(Y)\}\,, \tag{13}$$

where ${\rm Tr}_2$ stands for the trace operations with respect to the other half of the EPR pair.

To calculate the conditional output state of the other half of the EPR pair, we use the expressions of the coherent state in the momentum and position basis to give the wave function of the quadrature components of the coherent state:

$$\begin{aligned} {}_{\rm in}\langle X|\beta\rangle_{\rm in} &= \left(\frac{2}{\pi}\right)^{1/4} e^{-X^2+2\beta X-\frac{1}{2}|\beta|^2-\frac{1}{2}\beta^2}\,, \\ {}_1\langle Y|\gamma/\sqrt{2}\rangle_1 &= \left(\frac{2}{\pi}\right)^{1/4} e^{-Y^2+i\sqrt{2}\gamma Y-\frac{1}{4}|\gamma|^2+\frac{1}{4}\gamma^2}\,. \end{aligned} \tag{14}$$

Substituting eqs. (11), (9) and (14) into eq. (12) and integrating out the parameters $\beta$ and $\gamma$, eq. (12) becomes

$$\begin{aligned} \hat{\rho}_2(X,Y) &= \frac{(2/\pi)(1-\lambda^2)}{P(X,Y)} e^{-2(X^2+Y^2)} \int {\rm d}^2\alpha P(\alpha) f(\alpha) f^*(\alpha) \times \\ &\quad \times \hat{D}(\lambda[\alpha - \sqrt{2}(X-iY)])|0\rangle_2\langle 0|\hat{D}^{\dagger}(\lambda[\alpha - \sqrt{2}(X-iY)])\,, \end{aligned} \tag{15}$$

where $\hat{D}(\lambda[\alpha - \sqrt{2}(X-iY)])$ is a displacement operator and the function $f(\alpha)$ is

$$f(\alpha) = e^{-\frac{1}{2}|\alpha|^2} e^{\sqrt{2}\alpha(X+iY)} e^{\frac{1}{2}\lambda^2|\alpha-\sqrt{2}(X-iY)|^2}\,.$$

The probability distribution measured for observables $X$ and $Y$ is given by

$$P(X,Y) = (2/\pi)(1-\lambda^2)e^{-2(X^2+Y^2)} \int \mathrm{d}^2\alpha P(\alpha) f(\alpha) f^*(\alpha)\,. \tag{16}$$

We note that, for perfect squeezing of $\lambda = 1$, *i.e.*, for the ideal entangled EPR pairs, the probability of obtaining the observables goes to zero. This implies that after the measurement of Bell basis, no information of the input state can be obtained which is the necessary condition for faithful teleportation. We can easily prove that for all observables the probability is normalized:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}X\mathrm{d}Y\; P(X,Y) = 1\,. \tag{17}$$

According to the protocol of teleportation, the last step is to perform a unitary transformation on the other half of the EPR pairs with the above-measured results to mimic the input unknown state. In the experimental setup of ref. [15], this transformation is carried out by a beamsplitter which combines the other half of the EPR pair with a strong local oscillating coherent light modulated by the classical information of the measured values $X$ and $Y$. We denote the complex amplitude of this modulated intense coherent light as $M(X,Y) = |M(X)|e^{i\Phi(Y)}$, which is proportional to the classical outcomes of $X$ and $Y$, $M(X,Y) \propto X - iY$. When the beamsplitter has a negligible transmittance for the local oscillator and a high reflectance for the other half of the EPR pairs, the beamsplitter displays the displacement properties with a displacement equal to $Z = tM$ [29]. Taking $Z = \sqrt{2}g(X-iY)$, the state described by eq. (15) is transformed into

$$\begin{aligned}\hat{\rho}_2^{\mathrm{out}} &= \frac{(2/\pi)(1-\lambda^2)}{P(X,Y)} e^{-2(X^2+Y^2)} \int \mathrm{d}^2\alpha P(\alpha) f(\alpha) f^*(\alpha)\times \\ &\quad \times \hat{D}(\lambda\alpha + \sqrt{2}(g-\lambda)(X-iY))|0\rangle_2\langle 0|\hat{D}^\dagger(\lambda\alpha + \sqrt{2}(g-\lambda)(X-iY))\,,\end{aligned} \tag{18}$$

where $g$ represents a normalized classical gain for the transformation from the classical measured values $X$ and $Y$ to complex field amplitudes $Z$. Equation (18) expresses the output teleported state depending on the measured values of $X$ and $Y$.

Consider the case $g = 1$, $\lambda = 1$; we have $f(\alpha)f^*(\alpha) = e^{2(X^2+Y^2)}$ and we obtain the measurement probability from eq. (16) $P(X,Y) = (2/\pi)(1-\lambda^2)$. In this case, we get

$$\hat{\rho}_2^{\mathrm{out}} = \frac{(2/\pi)(1-\lambda^2)\int \mathrm{d}^2\alpha P(\alpha)|\alpha\rangle_2\langle\alpha|}{P(X,Y)} = \int \mathrm{d}^2\alpha P(\alpha)|\alpha\rangle_2\langle\alpha| = \hat{\rho}_{\mathrm{in}}\,. \tag{19}$$

It shows that the input unknown quantum state is perfectly teleported to the receiving station under the limit of infinite squeezing and unity classical transformation gain for the lossless measurement system. Thus at the ideal case the above three steps ensure that the unknown input quantum state is perfectly teleported [30].

*Fidelity of teleportation for coherent state.* – For the case of $\lambda < 1$ which corresponds to the nonideal EPR pair (*i.e.* imperfect squeezed state), the output state is a conditional output state of particular measurement outcomes, and the fidelity of teleportation system has to be discussed. When we ignore the outcomes and many teleportations take place, the resulting averaged output state behaves like a mixture of the unnormalized density matrix elements [8]

$$\begin{aligned}\hat{\rho}_T &= (2/\pi)(1-\lambda^2)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}X\mathrm{d}Y e^{-2(X^2+Y^2)} \int \mathrm{d}^2\alpha P(\alpha) f(\alpha) f^*(\alpha)\times \\ &\quad \times \hat{D}(\lambda\alpha + \sqrt{2}(g-\lambda)(X-iY))|0\rangle_2\langle 0|\hat{D}^\dagger(\lambda\alpha + \sqrt{2}(g-\lambda)(X-iY))\,.\end{aligned} \tag{20}$$

In eq. (20) we have used the normalized relation of eq. (17).

Indeed when $\lambda < 1$ and $g \neq 1$ eq. (20) shows that the output state overlaps with the input state partially. The extent of similarity between the output state and the input state $|\varepsilon\rangle$ is quantitatively evaluated with the fidelity $F$ [31]:

$$F = \langle\varepsilon|\hat{\rho}_T|\varepsilon\rangle\,. \tag{21}$$

For an input coherent state with the complex amplitude $\varepsilon$, we have $P(\alpha) = \delta(\varepsilon - \alpha)$. Using eq. (20) the average fidelity can be directly given from the definition of eq. (21) by integrating on $X$ and $Y$ from $-\infty$ to $\infty$:

$$F = \frac{1-\lambda^2}{1+g^2-2g\lambda} e^{-(1-g^2)\frac{1-\lambda^2}{1+g^2-2g\lambda}|\varepsilon|^2}\,. \tag{22}$$

Setting $\lambda = 0$, which is the classical limit of entanglement discussed in Part 2, and $g = 1$, the classical teleportation limit of $F = 1/2$ for a coherent state is obtained. When the fidelity $F > 1/2$, some quantum information of the input state is reconstructed in the outcome of the receiving station. The expression of fidelity is in agreement with the result calculated with the Wigner function [5]. It is obvious that the degree of entanglement as well as the classical gain are the important parameters for improving the fidelity of teleportation. When both the quantum entanglement $\lambda$ and the classical gain $g$ approach unity, the fidelity reaches the perfect value of 1. From eq. (22) we can see that for the finite squeezing $\lambda < 1$ and classical gain $g < 1$ the fidelity exponentially decreases along with increasing $|\varepsilon|^2$ which is proportional to the intensity of the input fields, thus it is impossible to do a successful teleportation of quantum input state of light with higher intensity, but when $g = 1$ or $\lambda = 1$, the fidelity will be independent of $|\varepsilon|^2$. As is well known, in experiment $g = 1$ is easier to be reached while $\lambda = 1$ is difficult to be realized, therefore, in the final step of teleportation, one should make $g = 1$ with great effort for performing the displacement transformation based on the classical measurement results.

In conclusion, we have analyzed the quantum teleportation of continuous variables using the theory of quantum-mechanical measurement in the general case of $\lambda < 1$ and $g \neq 1$ in the Schrödinger picture. The evolution of the quantum state during the teleportation process is obviously pointed step by step. It is shown that the quantum teleportation for a continuous variable can be characterized as reversible quantum operations. According to the state evolution equation, the fidelity for coherent states is deduced directly from the original definition. A useful result for this analysis is that the presented approach can be extensively used to deal with a variety of input states, especially squeezed states which have been discussed in the Heisenberg picture [32]. To our scheme, when the input state is a squeezed state, the input density operator should be replaced by $\hat{\rho} = |\alpha,\zeta\rangle\langle\alpha,\zeta|$; here $\alpha$ is the coherent complex amplitude, $\zeta$ is the squeezing parameter of the squeezed state, then, following the above-mentioned evolution the results for the given state can be deduced, and we are going to discuss the problem more extensively in a separate paper. We think that the new approach provides a fresh discussion to the theory of teleportation and might be of general interest.

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